Problem Set IV: Due Wednesday, November 23, 2016

1.) What is the relation between the volume and pressure of a gas of point particles moving within a cubic box of side *L*? Assume the walls are hard, so particles reflect elastically, and that two opposite sides move together or apart *slowly*. Comment on the relation between your result and those of thermodynamics.

2.) In the course so far, we have discussed *three* types of multiple time scale approximation methods in mechanics. Give a concise summary of these. Include a table. Your summary should include:

- a listing of the disparate time scales
- the approximation made
- why it works – what is the leverage
- any key features – i.e. resonances, etc.
- the 'canonical example' for each
- a one line summary of the bottom line for the canonical example

N.B.: For those who are puzzled at this sort of assignment, see: "The Innovators", by Walter Isaacson; location 1626 re: Grace Hopper.

3.) Determine the stable equilibrium positions for a simple pendulum which oscillates:

a.) horizontally, with $x = x_0 \cos \omega t$

b.) in a circle, with $x = r_0 \cos \omega t$, $y = r_0 \sin \omega t$.

Take $\omega >> \sqrt{g/\ell}$ and consider the full range of parameters.

4.) Now again consider a simple pendulum with support oscillating at $y = y_0 \cos \omega t$. If the pendulum has length ℓ (so $\omega_0 = \sqrt{g/\ell}$) and $\omega = 2\omega_0 + \in$, determine the conditions for, and growth rate of, parametric instability.

5.) Compute the threshold for parametric instability in the presence of linear frictional damping, as well as mismatch. For what range of mismatch $\in$ will instability occur?

6.) Let $H(q,p,t) = H_0(q,p) + V(q)\,d^2A/dt^2$ where $A(t)$ is periodic, with period $\tau << T$. Here T is the period of the motion governed by H_0.

a.) Derive the mean field (i.e. short time averaged) equations for this system.

b.) Show that these mean field equations may be obtained from the effective Hamiltonian

$$K(p,q) = H_0(p,q) + \frac{1}{4m}\left\langle \left(\frac{dA}{dt}\right)^2 \right\rangle \left(\frac{\partial V(q)}{\partial q}\right)^2 .$$

Here < > means a short time average. You may assume $H_0 = p^2/2m + V_0(q)$.

7.) FW 6.18

8.) FW 4.3 a-c

9.) FW 4.2